IMPLEMENTATION OF THE EIT RECONSTRUCTION TECHNIQUE BY USING THE GAUSS-NEWTON MINIMIZATION IN COMBINATION WITH A FINITE ELEMENT METHOD

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Abstract. Electrical impedance tomography (EIT) is widely studied when dealing with multiphase flow. This method presents a technical advantage over others as a imaging technique, since it is both a non-intrusive and radiation-free, with high temporal resolution. It allows to identify specific flow patterns, flow measurements, along with others applications. Basically, it consists of setting electrodes in the boundaries of a body to inject an electrical current to an electrical field to be formed and therefore to obtain electrical measurements such as voltage through these sensors. These measurements together with mathematical models allow to reconstruct an image of the interior due to the conductivity difference between two differents phases. This image is calculated by the algorithms of inverse problems, such as Gauss-Newton minimization regularized with the Tikhonov technique. In this work, it is implemented the direct problem with the Finite Element Method and the Gauss-Newton inverse problem to reconstruct the conductivity distribution of a domain and the results are compared with the software Electrical Impedance Tomography and Diffuse Optical Tomography Reconstruction Software (EIDORS).

Keywords: Electrical impedance tomography, imaging, EIDORS, Inverse problem, Finite element method

1. INTRODUCTION

The electrical impedance tomography (EIT) is a technique that is widely studied nowadays. Through the application of electrical current in adjacent pattern, it is possible to obtain a reconstructed image of the conductivity or permissivity. In this image, due to the differences in

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the conductivity or permissivity in the media, one could visualize the flow pattern, identifying different phases or bodies in the domain in question. The application of the electrodes is non-intrusive and low-cost, which makes this technique extremely attractive. Those techniques are employed in biomedical applications (Bodenstein et al., 2009) and industrial applications (Sharifi and Young, 2013). Specifically in industrial applications, it has been appreciated in different types of multiphase flow, as ascending and horizontal flows (Parvareh et al., 2010), different purposes as flow measurement (Meng et al., 2010) and phase distribution monitoring (Kourunen et al., 2011).

The data acquisition is realized through the use of electrodes around the surface of the domain, normally between 16 and 32 electrodes. In an application which the reconstruction is planar, the electrodes are all in the same plane, however if the reconstruction must be tridimensional, various planes of electrodes are set. An electrical current is injected within the domain through the first pair of electrodes and an electrical field is formed in this domain. In that manner, the corresponding electrode measures and register the voltage in a computer. This is done until all electrodes of the pattern are used as working electrode. A specific number of measurements is registered in that manner. These measurements are then utilized in the inverse problem (Graham, 2007).

In this work, a reconstruction software is implemented and compared with the software Electrical Impedance Tomography and Diffuse Optical Tomography Reconstruction Software (EIDORS). The software uses the finite element method for the direct problem and the regularized Gauss-Newton to solve the inverse problem.

2. MODELLING THE PROBLEM

2.1 The forward problem

Let be a domain Ω with a conductivity σ with a potential u on the electrodes which are applied a current. The equation that represents the phenomena is described as (Holder, 2004):

$$\nabla . (\sigma \nabla u) = 0, \qquad x \in \Omega \tag{1}$$

In order to express completely the model, boundary conditions must be set. In this work, the Complete Electrode Model is used, which involves the consideration of a contact impedance in the electrodes. Thus,

$$u + z_l \sigma \frac{\partial u}{\partial n} = U_l, \qquad x \in e_l, \qquad l = 1,2,\dots,L \tag{2}$$

$$\int \sigma \frac{\partial u}{\partial n} dS = I_l, \qquad x \in e_l, \qquad l = 1,2,\dots,L \tag{3}$$

$$\sigma \frac{\partial u}{\partial n} = 0, \qquad x \in \partial\Omega \backslash \bigcup_{l=1}^{L} e_l \tag{4}$$

where u is the potential distribution, n is the normal of the boundary $\partial\Omega$, z_l is the contact impedance, dS is the infinitesimal area and L is the number of electrodes.

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Additionally to those conditions, in order to ensure existence and the uniqueness of the solution, it is necessary to add the model the law of conservation of charge

$$\sum_{l=1}^{L} I_l = 0 \tag{5}$$

And to set a reference point

$$\sum_{l=1}^{L} U_l = 0 \tag{6}$$

2.2 The finite element method

Firstly, in order to apply the method, the domain Ω is fractionated in simplices, turning the domain into a finite number of elements Ω_k. In two dimensions a simplex is a triangle. The mesh is then formed by the sum of those simplices, which will have k simplices and N vertices, which are called nodes. To approximate the potential on the mesh, it is used functions which are linear on each simplex, and continuous across the face, called basis function. After applying the weak form to Equation (1), a system of equation must be solved (Holder, 2004):

$$AU = I \tag{7}$$

where $I = (0, I)^T$, $0 \in \mathbb{R}^{1xN}$, $I = (I_1, I_2, \dots, I_L)^T \in \mathbb{R}^{1xL}$ is the vector containing the injected currents. The matrix A is formed by

$$A = \begin{pmatrix} B & C \\ C^T & D \end{pmatrix} \tag{8}$$

where

$$B(i,j) = \int_{\Omega} \sigma \nabla \phi_i \cdot \nabla \phi_j \, dxdy + \sum_{l=1}^{L} \frac{1}{z_l} \int_{e_l} \phi_i \phi_j \, dS \quad i,j = 1,2,\dots,N \tag{9}$$

$$C(i,j) = -\frac{1}{z_l} \int_{ej} \phi_i \, dS \quad i = 1,2,\dots,N, j = 1,2,\dots,L \tag{10}$$

$$D(i,j) = \begin{cases} 0 & i \neq j \\ \dfrac{|e_j|}{z_j} & i = j \end{cases} \quad i,j = 1,\dots,L \tag{11}$$

And ϕ is the basis function and $|e_j|$ is the length in 2D of the electrode.

2.3 The measurement protocol

In order to express the Neumann boundary condition to the system, one should adopt a protocol of injection and acquisition of measurements. In an EIT equipment, different strategies to inject current have been proposed in the literature. The adjacent protocol is the most common

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pattern. The first pair of electrode injects a current into the system. The successive pairs of electrodes that are not currently injecting proceeds measuring the voltage. The injection continues to the next pair of electrode and then to the respective remaining pairs measure the potential until all measurements are finished. The procedure for an 8 electrodes case is described and can be seen in Fig. 1.
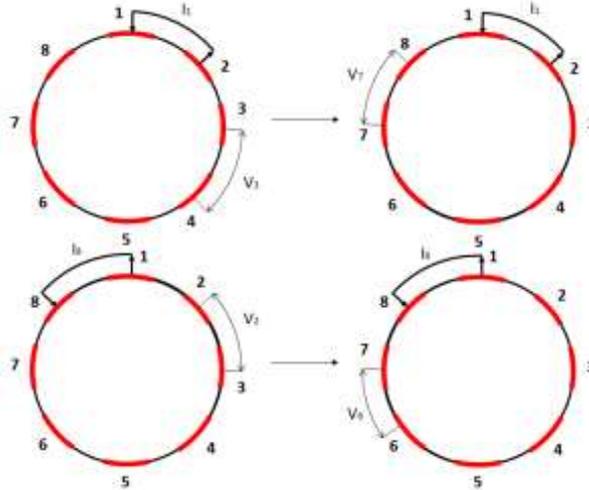
Figure 1. Schematic explaining the adjacent pattern.

Firstly, the current is injected in the pair 1 of electrode and measured in the pair 3, as the second electrode do not measure. It continues to measure in the subsequent pairs. The injected electrode is changed to the next and as it reaches to the 8 electrode pair, the measurement proceeds from electrode pair 2 until it reaches the last measuring electrode pair, the pair 6.

For the first the inject current vector $I_1 = [1, -1, 0, ..., 0]^T$, where $I_c = [I_1, I_2, ..., I_L]^T \in \mathbb{R}^{LxL}$ and the measurement matrix $M_{1,1} = [0,0,1, -1,0, ..., 0]^T$ where for each current pattern $M_{p,1} = [M_{1,1}, M_{1,2}, ..., M_{1,M}]^T \in \mathbb{R}^{LxM}$ where M represents the number of measurements.

2.4 The inverse problem

Consider an equation that has an input parameter and, as this parameter is chosen, an output is given. The inverse problem is to do the exact opposite path. To obtain of the parameter given the output. In the case of EIT, the problem is to obtain the conductivity parameter given the potential field. This problem is ill-posed and non-linear.

An ill-posed problem is defined when at least one of three conditions are not satisfied: existence, uniqueness, stability. To the solution to be well-posed a solution must exist, the solution must be unique and the solution must not have large variations when small changes.

To obtain a solution, one must minimize $\|Z - V\|_2^2$, where Z is the observations and V is the forward calculations (Lionheart, 2004). To overcome the ill-posedness, the equation must be regularized. The Tikhonov regularization is choise in a way that the term that must be minimized becomes

$$F(\sigma) = \|Z - V\|_2^2 + \lambda^2 \|L(\sigma - \sigma^*)\|_2^2 \tag{12}$$

where λ is the regularization term, L is the regularization matrix, σ is the estimated conductivity and σ^* is the prior information of the conductivity.

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Applying a linearization, the sigma vector can be obtained as

$$\delta\sigma = (J^T J + \lambda^2 L^T L)^{-1}(J^T(Z - V) + \lambda^2 L^T L(\sigma^* - \sigma)) \tag{13}$$

where J is the jacobian.

The election of the reconstruction parameter λ, also called of hyperparameter, is usually done manually, although a more comprehensive mean is available in (Graham, 2007). The prior matrix, L, the most common choices are the identity matrix, called the Tikhnov prior, and the NOSER prior, which corresponds to a diagonal with its entries as $(J^T J)^{factor}$, where the factor is usually 0.5 (EIDORS).

2.5 Jacobian

When using methods based on optimization it is common the use of Jacobian in the model. It is basically the calculation of the derivative of the voltage measurements based to the conductivity parameter. It could be also called of sensitivity maps as the rows of the matrix compute the sensitivity of the media. There are two forms to compute the Jacobian. The one way to compute the Jacobian that is more optimized, thus having the least computation time is the measurements fields (Polydorides, 2002), which is

$$J_{(d.m,n)} = \frac{\partial U_l^{(d,m)}}{\partial \sigma_n} = -\iint_{\Omega_n} \nabla u(I^d) \cdot \nabla u(I^m) dx dy \tag{14}$$

where m is the measurement and d is the current pattern, n is the simplex. Thus, to evaluate the Jacobian, one must obtain from the finite element method the value both gradients of the basis function multiplied by the electric potentials solved when using the current pattern and the electric potential solved when using the measurement field as explained in the Section 2.3.

3. METHODOLOGY

In this work, a EIT software is implemented and the finite element method is chosen to discretize the Eq. (1) to Eq. (6). In that manner, a mesh is required to perform the calculation over the domain. The domain consists in a circular domain with a 15 cm diameter and 16 electrodes equally spaced. The script was tested for both simulated data and experimental data. The mesh for the direct problem when using the simulated data and the inverse problem mesh are shown in Fig. 2. The direct problem mesh contains 1403 nodes and 2740 elements while the inverse problem is coarser to avoid inverse crime (Lionheart, 2004) with 722 nodes and 1394 elements. The results are all done with a injecting current of $1\ mA$, the contact impedance of $0.1\ \Omega \cdot m$. The prior matrix is the NOSER and the hyperparameter varied. A equipment constructed with a similar structure as (Dickin, 1996) was assembled and used to gather experimental data in order to test both the software and EIDORS implementation.

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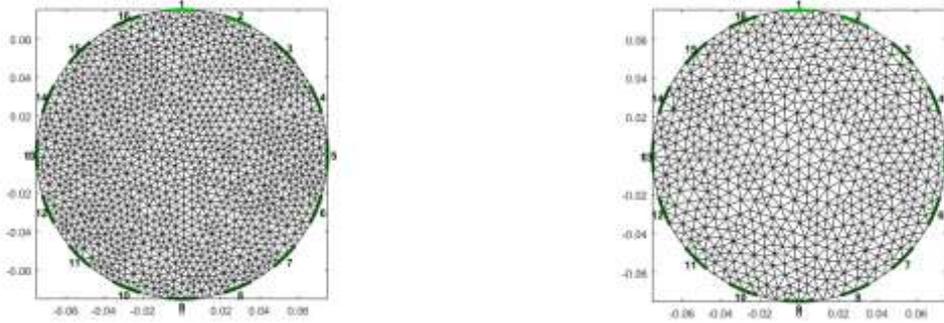
Figure 2. Mesh utilized for the direct problem and inverse problem.

4. RESULTS AND DISCUSSION

The algorithm is tested by the application of two synthetic phantoms and two experimental phantoms. Firstly, the direct problem is solved and validated through the comparison of the direct problem solved in EIDORS. The equation 1 to equation 6 are solved through the use of the finite difference method in the mesh cited in section 3. To validate the software, the measurements array from EIDORS, a consagrated software, is compared with the values of the software implemented, Fig. 3. It can be seen that the presented software has a good match with EIDORS, which validates its results. The voltage value in the surface of the 1° and 9° injections are shown in the surface plot in Fig. 4.
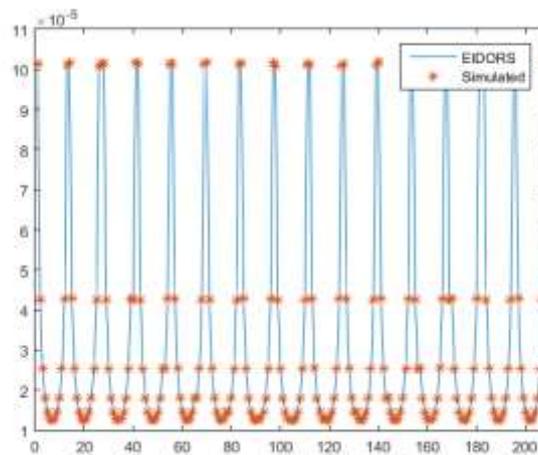
Figure 3. Measurement of EIDORS and simulated by the designed software.

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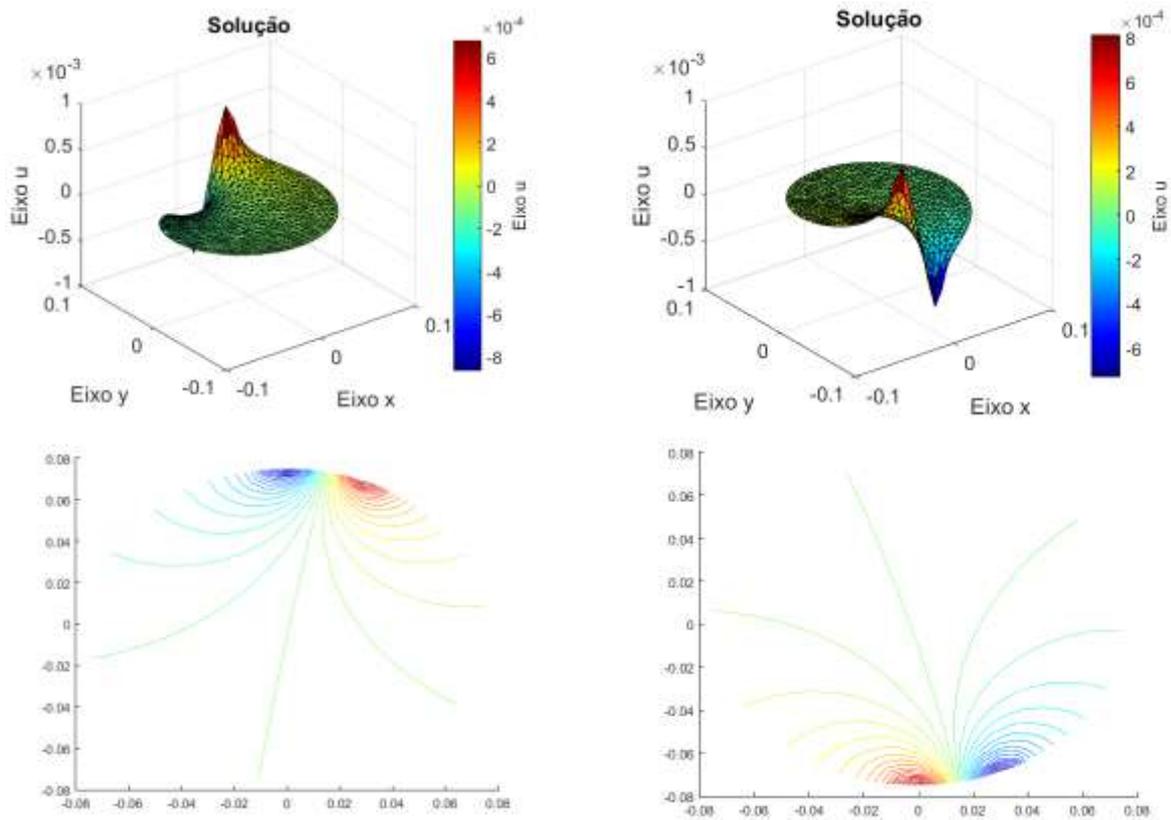
Figure 4. Results for the domain and contour plots.

Fig. 4 shows the voltage distribution, in Volts, in the circular domain. The blue area represents the electrode where the injected current sink while the red are represents the current being injected. This injection is responsible for the electrical field in the domain. The contour plot represent the electric field excited through the induced current.

The jacobian is calcu`led using equation 14. The computation of its elements is done by firstly calculating the voltage distribution by setting the injected current boundary condition according to the driving pattern, $U_d \in \mathbb{R}^{N+LxL}$, calculating the voltage distribution by setting the injected current boundary condition according to the measure pattern, $U_m \in \mathbb{R}^{N+LxLxM}$. The measurement and injection matrices structure are explained in section 2.3. The integration must be computed similar to the equation 9, in which the 9 entries of the 3x3 matrix resulted is put in the right position in the global matrix. This is done for each element in the mesh, resulting in a global matrix with no more than 9 entries. By multiplying the matrix of the integration by U_d and U_m in each element, it is obtained the row of the jacobian. By doing the same to each elements, the jacobian is finally assembled row by row, in which $J \in \mathbb{R}^{N \cdot MxNel}$ where Nel is the number of elements in the mesh.

Each row in the jacobian matrix corresponds to the sensitivity distribution inside the domain. The sensitivity distribution for the injection 1-2 in relation to the measurements 3-4 and 9-10 is shown in Fig. 5. It can be notice that disturbs in the conductivity near the electrodes which are measuring or injecting leads to a greater variation in the voltage. The sensitivity to conductivity changes in the interior points is lower compared to near the sensing and injecting electrodes. Although the latter is true, as near as the measuring sensor is to the injecting electrode, the greater the sensitivity in the interior.

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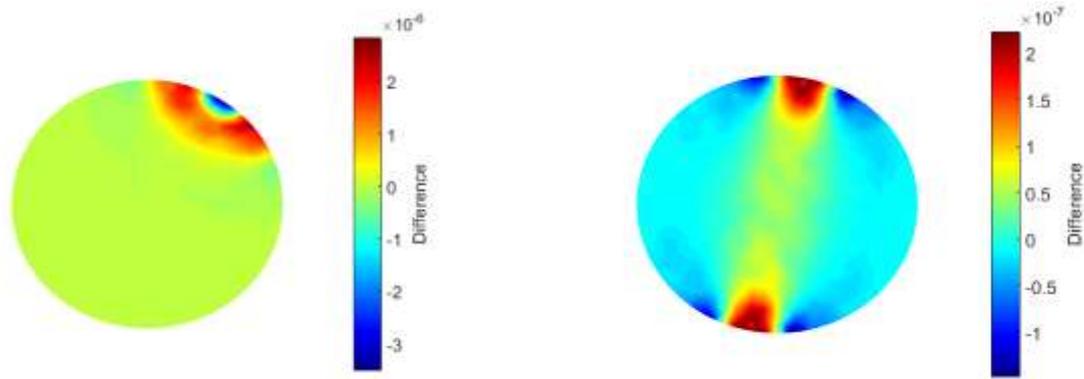
Figure 5. Sensitivity of the injection 1-2 related to the measurement 3-4 and 9-10 respectively.

The minimization procedure is done by the gauss-newton method, where its means of calculation is summarize in equation 13. To test the quality of the reconstruction, some synthetic results are obtained through the use of a different conductivity in certain region. A inclusion were included in the field of conductivity and the corresponding measurement signal was calculated using a finer mesh. Its result was then used as a measurement to minimize the conductivity. It is important to emphasize that it was incorporated a noise to the measurement (Lionheart, 2004). The conductivity of the domain is $1\,\text{S/m}$ while the inclusion has a conductivity of $0.01\,\text{S/m}$ in the case of the lower conductivity and $100\,\text{S/m}$ in the case of the higher conductivity. The conductivity distribution are shown in Fig. 6 for different hyperparameter.
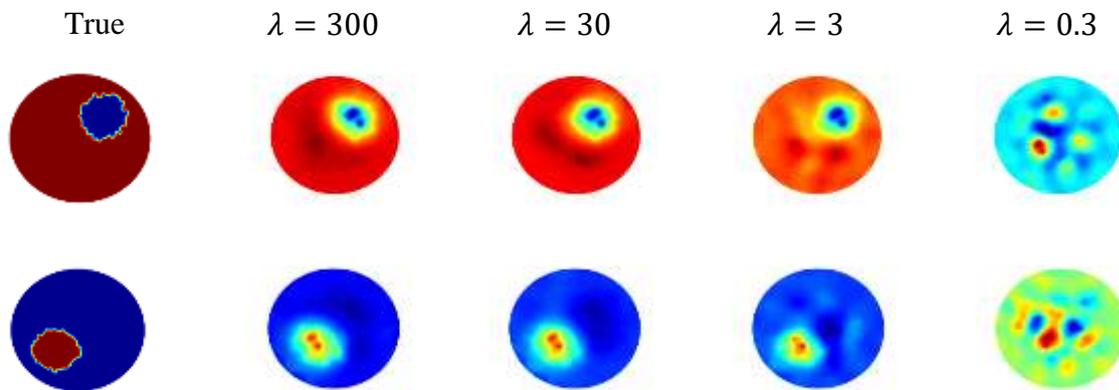
Figure 6. Simulated inhomogeneity estimated.

It is clear that the hyperparameter $\lambda = 0.3$, the reconstruction is dominated by the noise, thus it becomes difficult to reconstruct properly. For the subsequent choices, the algorithm is able to properly estimate. Although, if the hyperparameter is too high, there are increased smoothening, which is clearly unwanted.

It is also compared to the EIDORS software with experimental data. It is shown the results for a plastic filled with tap water domain with 16 equally spaced sensors made of copper in the Fig. 7. The region with blue color represents a lower conductivity while the region with red color represent the higher conductivity region. A plastic inhomogeneity is used in the first three

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phantoms while two metals inhomogeneity are used for the last one. It is seen that both softwares estimates reasonably well, although the EIDORS present a smoothier result.

Position	EIDORS	Software
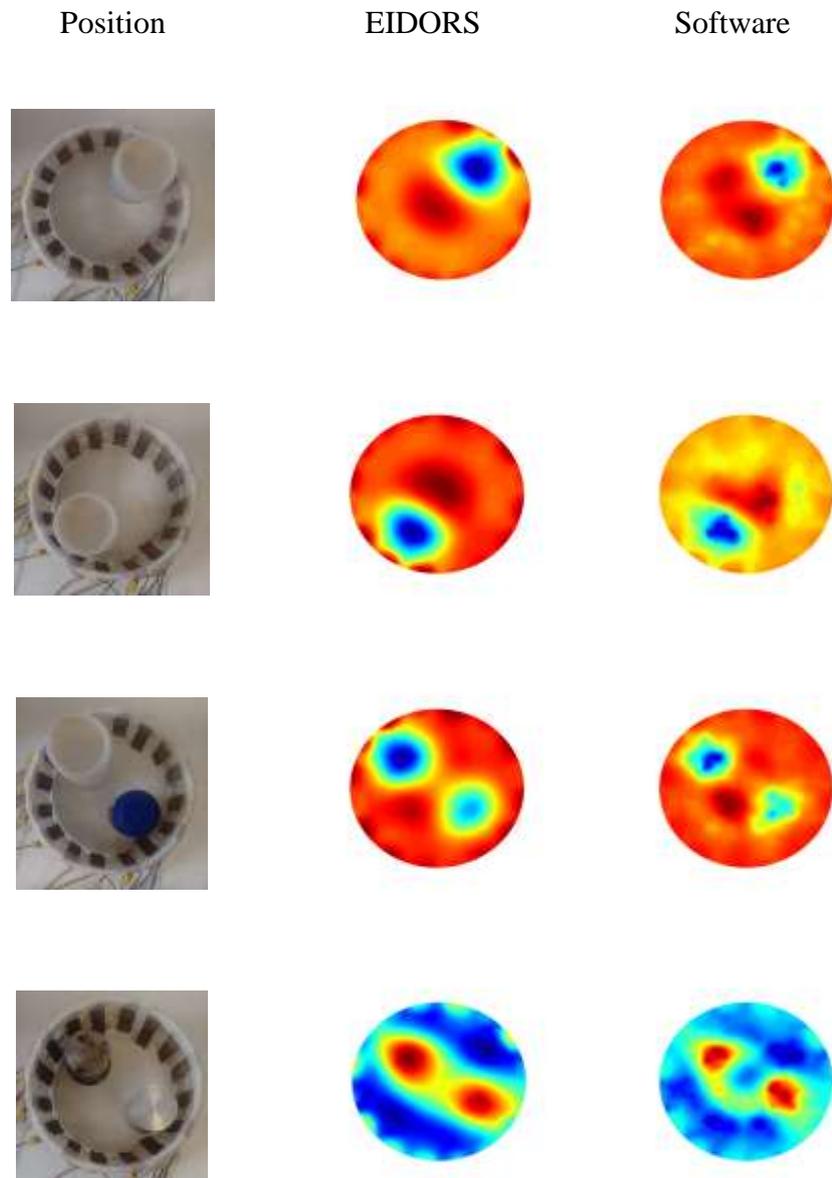
Figure 7. Experimental inhomogeneity estimated.

5. CONCLUSIONS

A software for the reconstruction of the conductivity in a domain based on the EIT technique is developed in Matlab environment and compared with the EIDORS software. The results shows that the direct problem, the complete electrode model solved with the finite element method, presented similar results as the EIDORS, which is a over validated software in the literature. The Gauss-Newton minimization, after computing the jacobian, was able to estimate the inhomogeneities of the phantom, although the EIDORS estimation is smoothier than the software developed.

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Acknowledgements

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