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ROBUST KALMAN FILTER FOR DESCRIPTOR SYSTEMS IN A DATA FUSION SCENARIO

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Abstract. *This paper deals with recursive robust estimation problem for descriptor systems in a data fusion scenario. This scenario occurs in cases where the system can operate under different failure conditions. Numerical examples are provided to show the effectiveness of the proposed algorithm.*

Keywords: *Robust, Descriptor System, Discrete-time, Kalman Filter, Data Fusion.*

1. INTRODUCTION

Descriptor systems (also known as singular systems or implicit systems) has been considered in some important applications related with economical systems Luenberger (1977), image modeling Hasan et al. (1995), and robotics Mills et al. (1989). Besides, the descriptor formulation contains the usual state-space system as a special case and can describe some dynamical systems for which state-space description does not exist.

State estimation of discrete-time descriptor systems have received great attention in the literature see, e.g., Ishihara et al. (2006). An interesting approach for descriptor Kalman filters is given in Ishihara et al. (2006), which develops a recursive robust filter via least-squares approach. This filter was created to operate with a unique measurement model. In situations where this unique measurement model suffers a failure, the state estimate can be impaired. Is possible to solve this problem considering more than one measurement model in state estimate with the data fusion approach.

In this paper is developed discrete-time robust Kalman filter for descriptor systems in situations that the system involve a multitude of measurement models. The robust filter is developed based on deterministic approach of the least square problems in data fusion scenario proposed in Sayed et al. (2000). An important issue of this approach is the guarantee of stability of the filter in online applications. Simulations was performed to show the effectiveness of the proposed robust filter.

2. PROBLEM STATEMENT

Consider the discrete-time descriptor system:

$$(E_{i+1} + \delta E_{i+1})x_{i+1} = (F_i + \delta F_i)x_i + w_i, \qquad i \geq 0, \tag{1}$$

$$z_{i,k} = (H_{i,k} + \delta H_{i,k})x_i + v_{i,k},\ k = 1, ..., L, \tag{2}$$

where $x_i \in \mathbb{R}^n$ is the state; $z_{i,k} \in \mathbb{R}^p$ are the measures output; $w_i \in \mathbb{R}^m$, and $v_{i,k} \in \mathbb{R}^p$ are the process and measurement noises; $E_{i+1} \in \mathbb{R}^{m\times n}$, $F_i \in \mathbb{R}^{m\times n}$, and $H_{i,k} \in \mathbb{R}^{p\times n}$ are the known system matrices, where E_{i+1} is a singular matrix.

The uncertainties δE_i, δF_i and $\delta H_{i,k}$ are modeled by:

$$\begin{aligned} \delta E_i &= M_i^f \Delta_{i,1} N_{i+1,e}, \\ \delta F_i &= M_i^f \Delta_{i,1} N_{i,f}, \\ \delta H_{i,k} &= M_{i,h}^{(k)} \Delta_{i,2} N_{i,h}^{(k)}, \end{aligned} \tag{3}$$

where M_i^f, $N_{i+1,e}$, $N_{i,f}$, $M_{i,h}^{(k)}$ and $N_{i,h}^{(k)}$ are known matrices. $\Delta_{i,1}$ and $\Delta_{i,2}$ are arbitrary matrices with $\|\Delta_{i,1}\| \leq 1$ and $\|\Delta_{i,2}\| \leq 1$.

Assuming $\{x_0, w_i, v_{i,k}\}$, initial condition and the process and measurement noises, are uncorrelated zero-mean random variables with second-order statistics:

$$cov\Big(\begin{bmatrix} x_0^T & w_i^T & v_{i,k}^T \end{bmatrix}^T\Big) = diag(P_0, Q_i\delta_{i,j}, R_{i,k}\delta_{i,j}) \tag{4}$$

where, $\delta_{i,j} = 1$, if $i = j$ and $\delta_{i,j} = 0$, if otherwise.

In this paper, it is proposed a solution for the problem of finding a recursive state estimation algorithm in a data fusion scenario for the uncertain descriptor system (2.). Given a sequence of measures output $\{z_{0,k}, z_{1,k}, ..., z_{i,k}\}$, where $k = 1, ..., L$, the main objective is to develop the best estimates to the state x_i.

For this, suppose that, at step i it is obtained an estimate for the state x_i, defined by $\hat{x}_{i|i}$, and there exists a positive-definite weighting matrix $P_{i|i}$ for the state estimation error $x_i - \hat{x}_{i|i}$, along with the new observation at time $i+1$, i.e., z_{i+1}. To update the estimate of x_i from $\hat{x}_{i|i}$ to $\hat{x}_{i+1|i+1}$, it is proposed the solution of following problem:

$$\min_{x_i, x_{i+1}} \max_{\delta E, \delta F, \delta H} \Bigg[||x_i - \hat{x}_{i|i}||^2_{P_{i|i}^{-1}} + ||(E_{i+1} + \delta E_{i+1})x_{i+1} - (F_i + \delta F_i)x_i||^2_{Q_i^{-1}} + \sum_{k=1}^{L} ||z_{i+1,k} - (H_{i+1,k} + \delta H_{i+1,k})x_{i+1}||^2_{R_{i+1,k}^{-1}} \Bigg]. \tag{5}$$

The solution of (5) is given in the next section.

3. ROBUST FILTERING FOR DISCRETE-TIME DESCRIPTOR SYSTEMS IN A DATA FUSION SCENARIO

The robust filter to be considered in this section is based on following least-squares approach:

Considered L uncertain models of the form

$$b_k := (A_k + \delta A_k)x + u_k, \quad k = 1, ..., L, \tag{6}$$

where u_k are measurement noises and δA_k are uncertainties associated to parameter matrices A_k.

The unknown parameter vector x is the same for all measurement b_k. This situation describes several distorted measurements of a unknown vector x arising from different uncertainty sources, as depicted in Fig. (1) of Sayed et al. (2000).

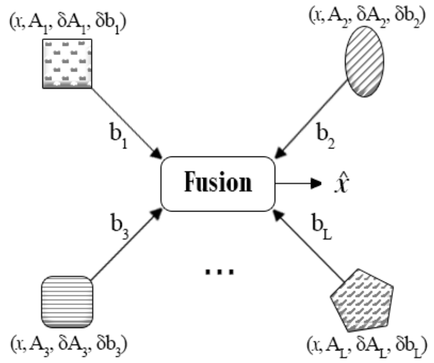

Figure 1- Data Fusion Scenario

Given the measurements in (6), the problem of estimating x optimally is solved by regularized robust least-squares approach in a weighted data fusion scenario describe in the next Lemma:

Lemma 3..1 *Sayed et al. (2000) Consider the problem of solving*

$$\hat{x} \;:=\; arg\min_{x}\max_{\delta A,\delta b}\left[||x||_Q^2 + \sum_{k=1}^{L} ||(A_k + \delta A_k)x - (b_k + \delta b_k)||_{W_k}^2 \right], \tag{7}$$

$$\text{where} \qquad \begin{bmatrix}\delta A_k & \delta b_k\end{bmatrix} := G_k \Delta \begin{bmatrix} N_{a,k} & N_{b,k}\end{bmatrix}. \tag{8}$$

where A_k are data matrices, b_k are the measurement vectors which are assumed to be known, δA_k and δb_k its uncertainties, respectively; x is the unknown vector, $x^T Q x$ is the regularization term and $Q = Q^T \geq 0$ and $W_k = W_k^T > 0$ are given weighting matrices. The solution of (7) is given by

$$\hat{x} := \left[\widehat{Q} + \sum_{k=1}^{L} A_k^T \widehat{W}_k A_k \right]^{-1} \sum_{k=1}^{L} \left[A_k^T \widehat{W}_k b_k + \hat{\lambda}_k N_{a,k}^T N_{b,k} \right]. \tag{9}$$

where the modified weighting matrices $\{\widehat{Q}, \widehat{W}_k\}$ computed from weighting matrices $\{Q, W_k\}$, as follow

$$\widehat{Q} \;:=\; Q + \sum_{k=1}^{L} \hat{\lambda}_k N_{a,k}^T N_{b,k}, \tag{10}$$

$$\widehat{W}_k \;:=\; W_k + W_k G_k (\hat{\lambda}_k I - G_k^T W_k G_k)^{\dagger} G_k^T W_k, \tag{11}$$

where, $\hat{\lambda}_k$ are non-negative scalars which $\hat{\lambda}_k > \|G_k^T W_k G_k\|$.

The robust Kalman filter for descriptor systems in data fusion scenario problem will be calculated based on the Lemma (3..1) according to the following Theorem:

Theorem 3..1 *Suppose that* $\left[\begin{array}{c} E_i \\ \bar{H}_i \end{array}\right]$ *has full-column rank and* $k = 1, ..., L$ *the quantity of measurement equations, for all* $i \geq 0$. *The robust filtered estimate in a data fusion scenario* $\hat{x}_{i|i}$, *resulting from (5), can be obtained from the following recursive algorithm:*

Step 0: (Initial Condition) If $M_{h,0}^{(k)} = 0$, *then*

$$\begin{aligned} P_{0|0} &:= (P_0^{-1} + \bar{H}_0^T \bar{R}_0^{-1} \bar{H}_0)^{-1}, \\ \hat{x}_{0|0} &:= P_{0|0} \bar{H}_0^T \bar{R}_0^{-1} \bar{z}_0. \end{aligned} \tag{12}$$

Where, $\bar{R}_0 := diag\left[R_{0,k}\right]$.

Else, find the scalar parameters $\lambda_{-1,k}$, *in the interval* $\lambda_{-1,k} > \|M_{h,0}^{(k)T} R_{0,k}^{-1} M_{h,0}^{(k)}\|$, *where*

$$\hat{R}_{0,k} := R_{0,k} - \hat{\lambda}_{-1,k}^{-1} M_{h,0}^{(k)} M_{h,0}^{(k)T}, \tag{13}$$

$$\bar{R}_0 := diag\left[\hat{R}_{0,k}\right], \tag{14}$$

$$\bar{N}_{h,0} := \begin{bmatrix} \sqrt{\hat{\lambda}_{-1,1}} N_{h,0}^{(1)} \\ \sqrt{\hat{\lambda}_{-1,2}} N_{h,0}^{(2)} \\ \vdots \\ \sqrt{\hat{\lambda}_{-1,L}} N_{h,0}^{(L)} \end{bmatrix}, \tag{15}$$

$$P_{0|0} := (P_0^{-1} + \bar{H}_0^T \bar{R}_0^{-1} \bar{H}_0 + \bar{N}_{h,0}^T \bar{N}_{h,0})^{-1}, \tag{16}$$

$$\hat{x}_{0|0} := P_{0|0} \bar{H}_0^T \bar{R}_0^{-1} \bar{z}_0. \tag{17}$$

Step 1*: If* $M_{f,i} = 0$ *and* $M_{h,i+1}^{(k)} = 0$, *then* $\hat{\lambda}_{i,k} := 0$. *Else, find the scalar parameters* $\hat{\lambda}_{-1,k}$, *in the interval*

$$\hat{\lambda}_{i,k} > \lambda_{n,i}^{(k)} := \left\| \begin{bmatrix} \sqrt{L} M_{f,i}^T & 0 \\ 0 & M_{h,i+1}^{(k)T} \end{bmatrix} \begin{bmatrix} \frac{1}{L} Q_i^{-1} & 0 \\ 0 & R_{i+1,k}^{-1} \end{bmatrix} \begin{bmatrix} \sqrt{L} M_{f,i} & 0 \\ 0 & M_{h,i+1}^{(k)} \end{bmatrix} \right\|. \tag{18}$$

Step 2*: If* $\hat{\lambda}_{i,k} \neq 0$, *replace the parameters* $\{Q_i, R_{i+1,k}, E_{i+1}, F_i, \bar{H}_{i+1}\}$, *by corrected parameters:*

$$\hat{\mathcal{Q}}_i := \begin{bmatrix} \mathcal{Q}_i & 0 \\ 0 & I \end{bmatrix}, where \quad \mathcal{Q}_i := \sum_{k=1}^{L} \frac{1}{L} \hat{Q}_{i,k}^{-1} \quad and \quad \hat{Q}_{i,k} := Q_i - \hat{\lambda}_{i,k}^{-1} M_{f,i} M_{f,i}^T, \tag{19}$$

$$\begin{aligned} \hat{\mathcal{R}}_{i+1} &:= \begin{bmatrix} \bar{R}_{i+1} & 0 \\ 0 & I \end{bmatrix}, where\, \bar{R}_{i+1} := diag\left[\hat{R}_{i+1,k}\right] and \\ &\quad \hat{R}_{i+1,k} := R_{i+1,k} - \hat{\lambda}_{i,k}^{-1} M_{h,i+1}^{(k)} M_{h,i+1}^{(k)T}, \end{aligned} \tag{20}$$

$$\hat{F}_i := \left[\begin{array}{c} F_i \\ \sqrt{\overline{\lambda}_i} N_{f,i} \end{array}\right], where\, \overline{\lambda}_i := \sum_{k=1}^{L} [\hat{\lambda}_{i,k}], \quad \hat{E}_{i+1} := \left[\begin{array}{c} E_{i+1} \\ \sqrt{\overline{\lambda}_i} N_{e,i+1} \end{array}\right], \tag{21}$$

$$\hat{H}_{i+1} := \left[\begin{array}{c} \bar{H}_{i+1} \\ \bar{N}_{h,i+1} \end{array}\right], where\, \bar{H}_{i+1} := \begin{bmatrix} H_{i+1,1} \\ H_{i+1,2} \\ \vdots \\ H_{i+1,L} \end{bmatrix} and\, \bar{N}_{h,i+1} := \begin{bmatrix} \sqrt{\hat{\lambda}_{i,1}} N_{h,i+1}^{(1)} \\ \sqrt{\hat{\lambda}_{i,2}} N_{h,i+1}^{(2)} \\ \vdots \\ \sqrt{\hat{\lambda}_{i,L}} N_{h,i+1}^{(L)} \end{bmatrix}. \tag{22}$$

Step 3: Update $\{P_{i|i}, \hat{x}_{i|i}\}$ *for* $\{P_{i+1|i+1}, \hat{x}_{i+1|i+1}\}$*, where*

$$P_{i+1|i+1} := \left(\begin{bmatrix} \hat{E}_{i+1} \\ \hat{H}_{i+1} \end{bmatrix}^T \begin{bmatrix} (\hat{\mathcal{Q}}_i^{-1} + \hat{F}_i P_{i|i} \hat{F}_i^T) & 0 \\ 0 & \hat{\mathcal{R}}_{i+1} \end{bmatrix}^{-1} \begin{bmatrix} \hat{E}_{i+1} \\ \hat{H}_{i+1} \end{bmatrix} \right)^{-1}, \tag{23}$$

$$\hat{x}_{i+1|i+1} := P_{i+1|i+1} \begin{bmatrix} \hat{E}_{i+1} \\ \hat{H}_{i+1} \end{bmatrix}^T \begin{bmatrix} (\hat{\mathcal{Q}}_i^{-1} + \hat{F}_i P_{i|i} \hat{F}_i^T) & 0 \\ 0 & \hat{\mathcal{R}}_{i+1} \end{bmatrix}^{-1} \begin{bmatrix} \hat{F}_i \hat{x}_{i|i} \\ \bar{Z}_{i+1} \end{bmatrix}. \tag{24}$$

where,

$$Z_{i+1} = \begin{bmatrix} \bar{z}_{i+1} \\ 0 \end{bmatrix}, \; where \; \bar{z}_{i+1} := \begin{bmatrix} z_{i+1,1} \\ z_{i+1,2} \\ \vdots \\ z_{i+1,L} \end{bmatrix}. \tag{25}$$

Proof 3..1 *The cost functional of the minimization problem (5) is rewritten as*

$$\begin{aligned} & \begin{bmatrix} x_i - \hat{x}_{i|i} \\ x_{i+1} \end{bmatrix}^T \begin{bmatrix} P_{i|i}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_i - \hat{x}_{i|i} \\ x_{i+1} \end{bmatrix} \\ & + \sum_{k=1}^{L} \left[\left(\begin{bmatrix} -(F_i + \delta F_i) & (E_{i+1} + \delta E_{i+1}) \\ 0 & (H_{i+1,k} + \delta H_{i+1,k}) \end{bmatrix} \begin{bmatrix} x_i - \hat{x}_{i|i} \\ x_{i+1} \end{bmatrix} - (b_k + \delta b_k) \right)^T \right. \\ & \qquad \left. \times \begin{bmatrix} \frac{1}{L} Q_i^{-1} & 0 \\ 0 & R_{i+1,k}^{-1} \end{bmatrix} \left(\bullet \right) \right] \end{aligned} \tag{26}$$

comparing (7) and (26), we obtain the following identifications

$$\begin{aligned} & x \leftarrow \begin{bmatrix} x_i - \hat{x}_{i|i} \\ x_{i+1} \end{bmatrix}, Q \leftarrow \begin{bmatrix} P_{i|i}^{-1} & 0 \\ 0 & 0 \end{bmatrix}, W_k \leftarrow \begin{bmatrix} \frac{1}{L} Q_i^{-1} & 0 \\ 0 & R_{i+1,k}^{-1} \end{bmatrix}, b_k \leftarrow \begin{bmatrix} F_i \hat{x}_{i|i} \\ z_{i+1,k} \end{bmatrix}, \\ & \delta b_k \leftarrow \begin{bmatrix} \delta F_i \hat{x}_{i|i} \\ 0 \end{bmatrix}, \delta A_k \leftarrow \begin{bmatrix} -\delta F_i & \delta E_{i+1} \\ 0 & \delta H_{i+1,k} \end{bmatrix}, A_k \leftarrow \begin{bmatrix} -F_i & E_{i+1} \\ 0 & H_{i+1,k} \end{bmatrix}, \\ & G_k \leftarrow \begin{bmatrix} \sqrt{L} M_{f,i} & 0 \\ 0 & M_{h,i+1}^{(k)} \end{bmatrix}, N_{a,k} \leftarrow \begin{bmatrix} -N_{f,i} & N_{e,i+1} \\ 0 & N_{h,i+1}^{(k)} \end{bmatrix} N_{b,k} \leftarrow \begin{bmatrix} N_{f,i} \hat{x}_{i|i} \\ 0 \end{bmatrix}, \\ & \Delta \leftarrow \begin{bmatrix} \frac{1}{\sqrt{L}} \Delta_{1,i} & 0 \\ 0 & \Delta_{2,i+1} \end{bmatrix}. \end{aligned} \tag{27}$$

According to (27) and (9), the optimum filtered estimate of x_{i+1} *in data fusion scenario,* $\hat{x}_{i+1|i+1}$*, is obtained with the solution of the following expression:*

$$\begin{aligned} \begin{bmatrix} \hat{x}_{i|i+1} - \hat{x}_{i|i} \\ \hat{x}_{i+1|i+1} \end{bmatrix} &= \begin{bmatrix} P_{i|i}^{-1} + \hat{F}_i^T \hat{\mathcal{Q}}_i \hat{F}_i & -\hat{F}_i^T \hat{\mathcal{Q}}_i \hat{E}_{i+1} \\ -\hat{E}_{i+1}^T \hat{\mathcal{Q}}_i \hat{F}_i & \hat{E}_{i+1}^T \hat{\mathcal{Q}}_i \hat{E}_{i+1} + \hat{H}_{i+1}^T \hat{\mathcal{R}}_{i+1}^{-1} \hat{H}_{i+1} \end{bmatrix}^{-1} \\ & \times \begin{bmatrix} -\hat{F}_i^T \hat{\mathcal{Q}}_i \hat{F}_i \hat{x}_{i|i} \\ \hat{E}_{i+1}^T \hat{\mathcal{Q}}_i \hat{F}_i \hat{x}_{i|i} + \hat{H}_{i+1}^T \hat{\mathcal{R}}_{i+1}^{-1} Z_{i+1} \end{bmatrix}. \end{aligned} \tag{28}$$

Pre-multipling both sides of (28) by $\begin{bmatrix} 0 \\ I \end{bmatrix}^T$*, it follows that:*

$$\hat{x}_{i+1|i+1} = \left[\begin{array}{cc} 0 & I \end{array}\right] \left[\begin{array}{cc} P_{i|i}^{-1} + \hat{F}_i^T \hat{\mathcal{Q}}_i \hat{F}_i & -\hat{F}_i^T \hat{\mathcal{Q}}_i \hat{E}_{i+1} \\ -\hat{E}_{i+1}^T \hat{\mathcal{Q}}_i \hat{F}_i & \hat{E}_{i+1}^T \hat{\mathcal{Q}}_i \hat{E}_{i+1} + \hat{H}_{i+1}^T \hat{\mathcal{R}}_{i+1}^{-1} \hat{H}_{i+1} \end{array}\right]^{-1} \times \left[\begin{array}{c} -\hat{F}_i^T \hat{\mathcal{Q}}_i \hat{F}_i \hat{x}_{i|i} \\ \hat{E}_{i+1}^T \hat{\mathcal{Q}}_i \hat{F}_i \hat{x}_{i|i} + \hat{H}_{i+1}^T \hat{\mathcal{R}}_{i+1}^{-1} Z_{i+1} \end{array}\right]. \quad (29)$$

After some algebra, the filtered estimate equation $\hat{x}_{i+1|i+1}$ *is given by (24) in Step* i *with the auxiliary variable* $P_{i+1|i+1}$*, defined as (23).*

4. NUMERICAL EXAMPLES

In this section was presented the numerical example, to show the effectiveness of the robust filter developed in this note, were considered the following parameters and uncertainties:

$$\begin{gathered} E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, F = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0.2 & 0.2 & 0.2 \end{bmatrix}, Q = \begin{bmatrix} 1.2 & 0 & 0 \\ 0 & 1.6 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \\ H_1 = \begin{bmatrix} 0.05 & 0.04 & 0.06 \end{bmatrix}, H_2 = \begin{bmatrix} 1.4 & 0.8 & 1 \end{bmatrix}, H_3 = \begin{bmatrix} 1.3 & 0.7 & 0.9 \end{bmatrix}, \\ R_1 = 0.18, R_2 = 0.16, R_3 = 0.17, \\ M_f = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1.3 \end{bmatrix}, N_f = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.13 \end{bmatrix}, \\ M_{h,1} = \begin{bmatrix} 0.8 & 0.8 & 0.8 \end{bmatrix}, M_{h,2} = \begin{bmatrix} 0.7 & 0.7 & 0.7 \end{bmatrix}, M_{h,3} = \begin{bmatrix} 0.6 & 0.6 & 0.6 \end{bmatrix}, \\ N_{h,1} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 20 \end{bmatrix}, N_{h,2} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 28.18 & 0 \\ 0 & 0 & 2 \end{bmatrix}, N_{h,3} = \begin{bmatrix} 2.1 & 0 & 0 \\ 0 & 28.19 & 0 \\ 0 & 0 & 2.1 \end{bmatrix}. \end{gathered} \quad (30)$$

The comparative study between the robust filter proposed in Ishihara et al. (2006) and the robust filter in a data fusion scenario is shown in Figure 2.

Notice that, the parameter matrix $H_{i,1}$ used in the robust filter of Ishihara et al. (2006) is simulating a modeling error. The root mean square (rms) error estimator was simulated. The curves were obtained from $i = 0, ...100$ for each recursive step of 4000 Monte Carlo simulations. Our estimator outperforms the estimator given in Ishihara et al. (2006) as can be seen in Figure 2.

Remark 4..1 *From (23) and (24), it is easy to verify that for descriptor systems without uncertainties (*$M_{f,i} = 0$*,* $M_{h,i+1} = 0$*,* $N_{e,i+1} = 0$*,* $N_{f,i} = 0$*,* $N_{h,i+1} = 0$*), this algorithm is the usual descriptor Kalman filter in a data fusion scenario:*

$$P_{i+1|i+1} := \left(\left[\begin{array}{c} E_{i+1} \\ \bar{H}_{i+1} \end{array}\right]^T \left[\begin{array}{cc} (Q_i + F_i P_{i|i} F_i^T) & 0 \\ 0 & \bar{R}_{i+1} \end{array}\right]^{-1} \left[\begin{array}{c} E_{i+1} \\ \bar{H}_{i+1} \end{array}\right] \right)^{-1} \quad (31)$$

$$\hat{x}_{i+1|i+1} := P_{i+1|i+1} \left[\begin{array}{c} E_{i+1} \\ \bar{H}_{i+1}^T \end{array}\right]^T \left[\begin{array}{cc} (Q_i + F_i P_{i|i} F_i^T) & 0 \\ 0 & \bar{R}_{i+1} \end{array}\right]^{-1} \left[\begin{array}{c} F_i \hat{x}_{i|i} \\ \bar{z}_{i+1} \end{array}\right] \quad (32)$$

where,

$$\hat{H}_{i+1} := \begin{bmatrix} H_{i+1,1} \\ H_{i+1,2} \\ \vdots \\ H_{i+1,L} \end{bmatrix}; \quad \bar{R}_{i+1} := \begin{bmatrix} R_{i+1,1} & 0 & \cdots & 0 \\ 0 & R_{i+1,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{i+1,L} \end{bmatrix}; \quad \bar{z}_{i+1} := \begin{bmatrix} z_{i+1,1} \\ z_{i+1,2} \\ \vdots \\ z_{i+1,L} \end{bmatrix}. \tag{33}$$

Remark 4..2 *From (23) and (24), it is easy to verify that for descriptor systems with ($k = 1$, a unique measurement model) this algorithm is the descriptor robust Kalman filter of Ishihara et al. (2006), Theorem $IV.1$.*

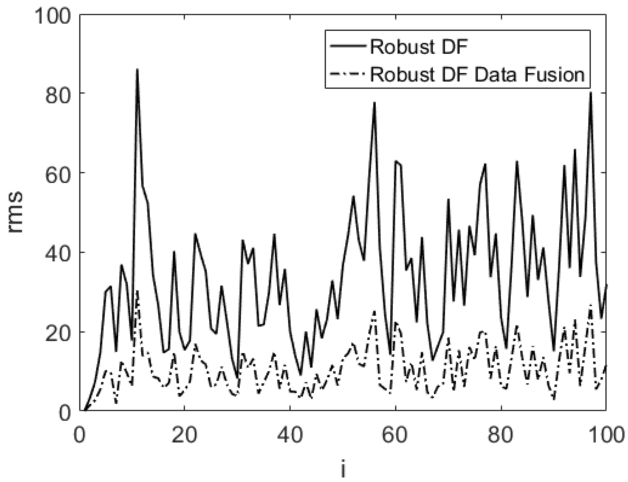

Figure 2- Comparison between Robust Filter of Ishihara et al. (2006) and Robust Filter in a data fusion scenario.

5. CONCLUSIONS

In this paper was developed the robust Kalman filter for descriptor systems in a data fusion scenario described in Theorem (3..1). The estimate was deduced based on the Lemma (3..1) proposed in Sayed et al. (2000). This filter is a important technique of estimation in situations that involve a multitude of measurement models. Numerical example showed the effectiveness of this approach.

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REFERENCES

Kalman, R. E. (1960), A new approach to linear filtering and prediction problems. Journal of basic Engineering, 82(1), 1440-1445.

Hasan, M. A.; Azimi-Sadjadi, M. R. (1995), Noncausal image modeling using descriptor approach. IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing, 42(8), 536-540.

Luemberger, D. (1977), Dynamic equations in descriptor form. IEEE Transactions on Automatic Control, 22(3), 312-321.

Mills, J. K.; Goldenberg, A. A. (1989), Force and position control of manipulators during constrained motion tasks. IEEE Transactions on Automatic Control, 5(1), 30-46.

Nikoukhah, R.; Willsky, A. S.; Levy, B. C. (1992), Kalman filtering and Riccati equations for descriptor systems. IEEE Transactions on Automatic Control, 37(9), 1325-1342.

Sayed, Ali H.; Al-Naffouri, T. Y.; Kailath, T. (2000), Robust estimation for uncertain models in a data fusion scenario. IFAC Proceedings Volumes, 33(15), 899-904.

Ishihara, J. Y.; Terra, M. H.; Campos, J. C. T. (2006), Robust Kalman filter for descriptor systems. IEEE Transactions on Automatic Control, 51(8), 1354-1354.

Silva, B. M. C. (2018), "*Robust Filtering for Linear Systems in a Data Fusion Scenario*", Master Dissertation, PPGMC/UESC, Ilhéus, BA.