XXI ENMC
ENCONTRO NACIONAL DE MODELAGEM COMPUTACIONAL
IX ECTM
ENCONTRO DE CIÊNCIA E TECNOLOGIA DE MATERIAIS

08 a 11 de Outubro de 2018
Instituto Federal Fluminense
Búzios - RJ

LYAPUNOV STABILITY IN A MODEL THAT DESCRIBES THE RADIAL MOVEMENT OF A PARTICLE IN ACOUSTIC LEVITATION

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Abstract. *The stability of the radial trajectory of a small sphere in a single-axis acoustic levitator was investigated. A mathematical model based on acoustic radiation forces and real parameters is proposed to describe the dynamics of the sphere radial movement. The stability of the motion was investigated via Lyapunov exponents diagram, showing a complex structure sharing different regions of chaos and regularity according to the model parameters.*

Keywords: *Lyapunov exponents, Acoustic levitation, Non-linear phenomena*

1. INTRODUCTION

Currently, the acoustic handling technique has received considerable attention in a wide range of areas as biology (Scheeline (2012)), analytical chemistry (Santesson (2004)), scattering of nanoparticles (Schenk et. al. (2012)), and others.

The most common type of acoustic levitator, called single-axis, consists of an ultrasonic transducer and a reflector separated by a multiple integer number of a half wavelength, producing a standing wave field in the air gap. In such a device, samples much smaller than the wavelength may be levitated by acoustic radiation forces that traps the samples at the pressure node. In this context, it has been observed that small samples in levitation do not remain static but oscillate in a particular movement around an equilibrium point (Andrade & Péres (2014)). These spontaneous oscillations are many time irregular and related to nonlinear effects that occasionally emerges.

In the present work, we are interested in investigating the nonlinear effects of acoustic radiation forces in the trajectories described by a small sphere levitating at the pressure node. The trajectory in the horizontal direction is obtained through a mathematical model and compared to experimental data. In addition, a stability diagram based on Lyapunov exponents is investigated and the results point out to a complex frontier between regions of chaos and regularity.

2. BACKGROUND

In the acoustic levitation process of a single-axis levitator, basically two radiation forces act on a single object. The primary radiation force in the axial direction, F_z, acts in the direction of propagation of the wave and is the responsible for counteracting the gravity force. Yosioka & Kawasima (1955) and Gor'kov (1962) calculated the acoustic radiation pressure on a sphere obtaining the the primary acoustic force,

$$F_z = 4\pi R^3 E_{ac} k \left[1 + \frac{3(\rho_p - \rho_m)}{2\rho_p + \rho_m} - \frac{\beta_p}{\beta_m} \right] \sin[2kz].,\tag{1}$$

where R is the sample radius, E_{ac} is the average acoustic energy density, k is the wave number and z is the distance from the nearest pressure antinode. The first brackets in Eq. (1) is called acoustic contrast factor, G. In such factor, β is the compressibility, ρ is the density, and subscripts p and m are used to indicate the particle and medium, respectively. For $G < 0$, samples move to the pressure antinodes by the action of the primary acoustic force. However, if particles are denser than the surrounding fluid ($G > 0$), they are driven to the pressure nodes.

The second force is a transversal component denoted as radial radiation force, F_x. This component is two orders of magnitude smaller than the axial component and is defined as,

$$F_x = \frac{V}{4} \left[\frac{\partial P^2(x)}{\partial x} \right] \left\{ B - \left[B + \left(1 - \frac{\beta_p}{\beta_m} \right) \right] cos^2\left[kz \right] \right\},\tag{2}$$

where $P(x)$ is the acoustic pressure amplitude which may depend on the radial position, x.

Forces F_z and F_x acting on a small sphere in an single-axis acoustic levitator are illustrated in Fig. 1.

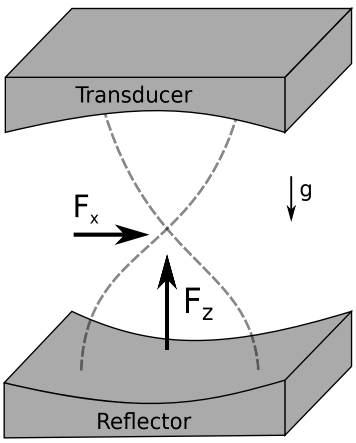

Figure 1- Axial radiation force, F_z, and radial radiation force, F_x, acting on a small sphere in a single-axis levitator.

3. RESULTS AND DISCUSSION

3.1 The Data Set

The data set used in this work was obtained by a single-axis acoustic levitator of a 20.3 kHz Langevin type transducer. Both the transducer and reflector have a concave radiating surface with a curvature radius of 35 mm and 33 mm, respectively (Andrade et. al. (2014)).

A small sphere is placed at the pressure node and its motion is recorded by a high-speed camera. The displacement to the center of the pressure node is then obtained for the positions in the radial direction x. Figure 2 shows a time series of positions in x-direction of a $4mm$ diameter glass sphere.

We also constructed the phase plane. Assuming an arbitrarily time-delay t_d in the time series, the phase plane is obtained in a $x(t)$ variable plotting: $x(t) \times x(t+t_d)$. Such procedure is called attractor reconstruction (Takens (1981)). In panel (B) of Fig. 2, the attractor reconstruction is shown for the time series (A). We observe that the trajectory in the x-direction seems to move in an irregular way, preferably visiting the neighborhood of the points $(1, 1)$ and $(-1, -1)$. Furthermore, it is interesting to note that the attractor of Fig.2 (B) is similar to chaotic attractors related in the literature.

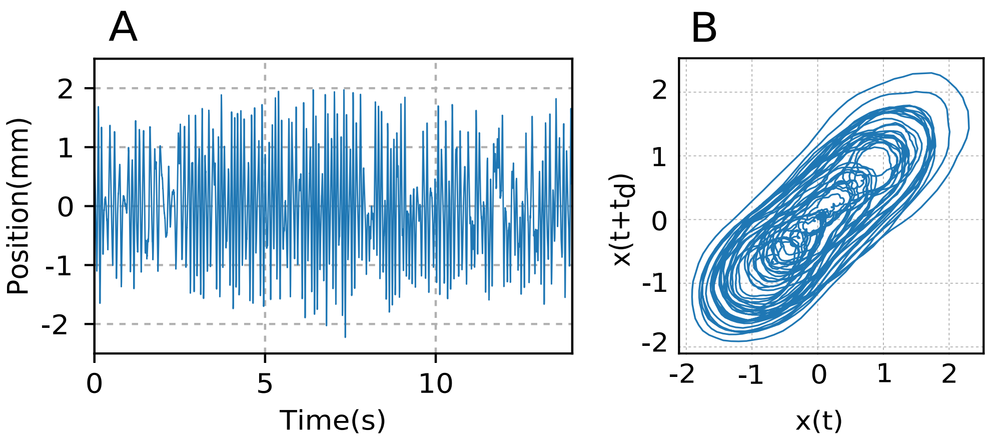

Figure 2- (A) Time series of the radial movement of glass sphere in levitation and (B) Attractor reconstruction of time series showed in (A) with $t_d = 1.2s$.

3.2 The Radial Model

In this section, we develop a theoretical model based on radial acoustic radiation force. The Newton second law in the radial direction gives:

$$m\frac{d^2x}{dt^2} + 6\pi\mu a\frac{dx}{dt} - \frac{V}{4}\left[B - \left[B + \left(1 - \frac{\beta_p}{\beta_m}\right)\right]\cos^2(kz)\right]\left(\frac{\partial P^2(r)}{\partial r}\right) = 0, \tag{3}$$

where the quantity, $6\pi\mu a\frac{dx}{dt}$, is the Stokes'drag force on a small sphere of radius a. Now, let us consider some approximations. Firstly, since the range of movement of the particle in the axial direction is small, we assume $cos(kz) \approx 1$. Furthermore, we consider the acoustic pressure amplitude as a zero-order Bessel Function truncated in the first non-linear term, $P(x) \approx (1 - x^2/4)$. Then, re-wrinting Eq. (3) and considering an external periodic force we have,

$$\frac{d^2x}{dt^2} + C_x\frac{dx}{dt} + \gamma_x\left(-x + \frac{1}{4}x^3\right) = D_x\cos(\omega_r t), \tag{4}$$

where,

$$C_x = 6\pi\mu a/m, \tag{5a}$$
$$\gamma_x = V(1 - \beta_p/\beta_m)/4m. \tag{5b}$$

Introducing an adimensional time $\tau = \omega_x t$, Eq. (4) is replaced by,

$$\frac{d^2x}{d\tau^2} + \hat{C}_x\frac{dx}{d\tau} + \hat{\gamma}_x\left(-x + \frac{1}{4}x^3\right) = \hat{D}_x\cos(\tau), \tag{6}$$

Equation (6) gives the movement of the suspended sphere in the radial direction, whose solution is showed in Fig. 3 concerning the following values to the parameters: $\hat{\gamma}_x = 1$, $\hat{D}_x = 0.95$ and $\hat{C}_x = 0.316$. The attractor is reconstructed in Fig. 3 (B) with $\tau_d = 12$.

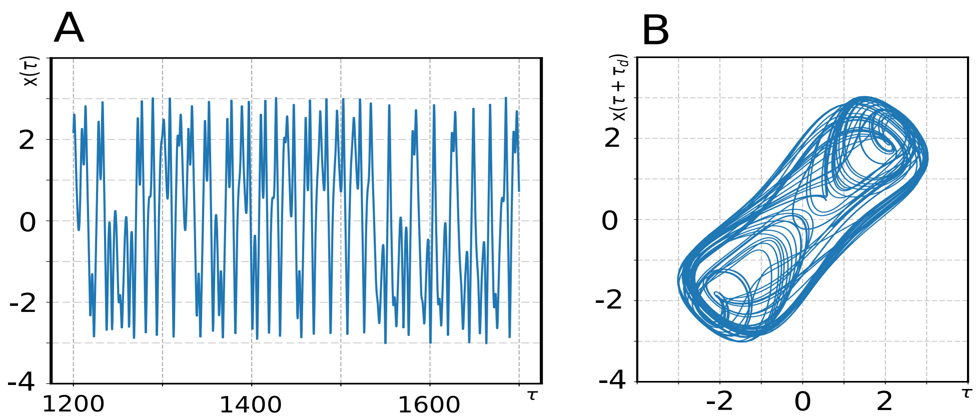

Figure 3- (A) Part of numerical solution of Eq. (6) with $\hat{\gamma}_x = 1$, $\hat{D}_x = 0.95$ and $\hat{C}_x = 0.316$. (B) Attractor reconstructed with $t_d = 12$

Under topological point of view, the analytical attractor of Fig. 3 (B) and the experimental attractor of Fig. 2 (B) are quite similar. In the next section, we investigate the stability of the solutions of Eq. 6 by means of Lyapunov exponents.

3.3 Lyapunov Stability

To evaluate the Lyapunov diagram for Eq. (6) we fixed $\hat{\gamma}_x = 1$ and we divided the space parameter $(\hat{C}_x, \hat{D}_x)$ into a grid of 800×800. In our simulation, for each point $(\hat{C}_x, \hat{D}_x)$ of Eq. (6) we computed the Lyapunov exponent according to the method described in (Sprott

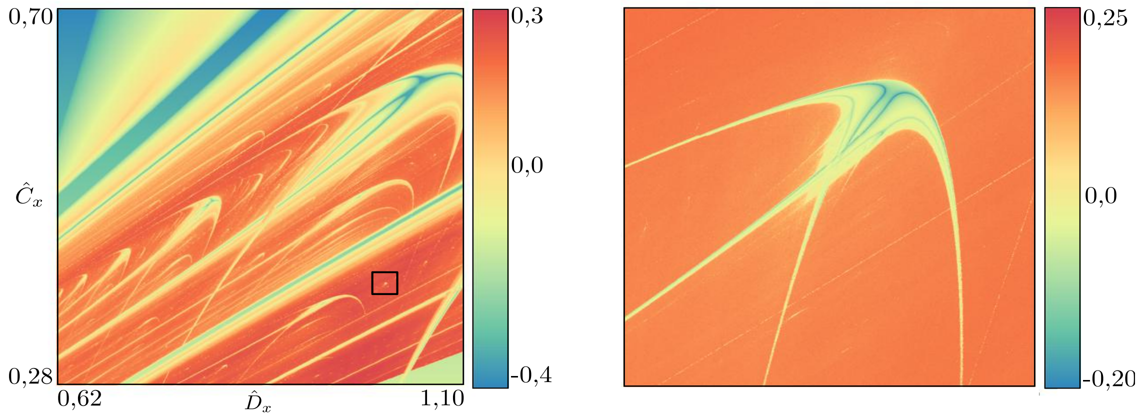

Figure 4- Lyapunov diagram of Eq. (6) for a grid of 800×800. Lyapunov exponents are posed in colors according to the pallete. The picture at right emphasizes a periodic window in a finer scale with $\hat{C}_x \in [0.38605 : 0.39655]$ and $\hat{D}_x \in [1.001; 1.0136]$.

(2003)), starting an arbitrarily initial condition. Each value for the Lyapunov exponent related to the point $(\hat{C}_x, \hat{D}_x)$ is displayed in a palette of colors as shown in Fig. 4.

In Fig. 4, it can be observed that the horizontal trajectory of a suspended particle may assume periodic ($\gamma < 0$), quasi-periodic ($\gamma = 0$) and chaotic behavior ($\gamma > 0$) according to the complexity structures presented in the diagram. Such structures present a fine frontier between chaotic and periodic motions and may exist in different scales as emphasized in the amplification of the figure.

4. CONCLUSIONS

We presented here some numerical investigations on the trajectory described, in the radial direction, by a small sphere in a single-axis acoustic levitator. This trajectory was obtained experimentally by a data set and a theoretical model for comparision. The time series generated by experimental data showed typical topologies of nonlinear systems, as a chaotic attractor in the horizontal movement. Such behavior was confirmed qualitatively by the mathematical model and numerical simulations. It was observed that the existence of chaotic attractors related to the horizontal movement depends on the presence of an external force and on the choice of the profile of the amplitude acoustic pressure. We further investigate the stability of the theoretical solutions by means of Lyapunov exponents in a parameter space, where we observed a complex diagram whose the related Lyapunov exponents indicate the presence of regular behavior and also chaotic ones.

Finally, it worth to mention that the study of how the nonlinear acoustic forces affects the particle stability (Lyapunov diagram) may help to comprehend how to minimize the amplitudes of the spontaneous oscillations.

Acknowledgements

The authors would like to thank the Brazilian agencies: CAPES/FAPEG (88881.127849/2016-01) and CNPq (458805/2014-9) for the financial support.

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