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COMPUTATIONAL METHOD $H\infty$ APPLIED TO DEXTEROUS HAND MASTER - DHM

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Abstract. *The most commonly used controller design techniques, such as LQR, PID and LQG, require only a simple model of the plant to be controlled. In particular, these control system design methods did not take into account, explicitly and quantitatively, possible perturbations, uncertainties, modeling errors in the system transfer function, noise measurements, among other uncertainties during the dynamic process of experiments. This work uses the H-infinite computational method for the robust control of the mechanical system of the DHM (Dexterous Hand Master) through the mathematical modeling of the DHM plant, in order to determine a suitable robust controller of this plant able to stabilize the system, minimizing the effects of disturbances that are inherent in the system. The H-infinite controller for this system is designed considering some modeling of the uncertainties that characterize the disturbances inherent in the DHM plant. The simulations are performed in MATLAB software and satisfactory results are presented.*

Keywords: *Computational Methods, $H\infty$, Robust, DHM Control*

1. INTRODUCTION

Controller design techniques, such as LQR, PID, and LQG, require only a simple model of the plant to be controlled, and it is generally important for the designer to have the following query: *the designed controller is robust against external uncertainties and perturbations ?*. These more traditional methods of control system design were created without explicitly and quantitatively taking into account these types of problems that affect the behavior of the system during the control process. In the literature for designing a PID controller with reasonably good performance, an accurate model of the plant to be controlled is not required. The PID controllers were considered to have some robustness in the sense of tolerating the uncertainty of the model. However, it should be noted that, in the design of the PID controllers, no quantitative information on the mismatch of the model is used. However, PID controllers may sometimes not guarantee system robustness.

The optimum control projected on Wiener's eminent filtering work in 1940 reached its maturity in 1960 with what we call the Linear Quadratic Gaussian or LQG control. Aerospace engineers were particularly successful in applying the LQG controller, but when other control engineers tried to use this same methodology in everyday industrial problems, a different problem arose. The accuracy of the plant models were often not available and the white noise hypothesis was not always relevant or significant in practice for control engineers. As a result, LQG projects sometimes did not guarantee robustness enough to be used in practice.

The $H\infty$ project played an important role in the study and analysis in control theory since its original formulation in an input-output configuration with (Zames, 1981). In this period, with the influential work of Zames, motivated by the shortcomings of the LQG control, there was a significant shift towards $H\infty$ optimization for robust control. $H\infty$ solutions in state-space form were strictly derived for the linear time-invariant case that required the resolution of several associated Riccati equations (Doyle et al., 1989). State-space formulas are derived for all drivers that solve a standard problem $H\infty$. The problem is reduced to a number $\gamma > 0$, find all controllers so that the $H\infty$ norm of the closed-loop transfer function is (strictly) less than γ. A controller exists if and only if the stabilizing solutions unique to the two Riccati algebraic equations are positive and the spectral radius of its product is less than γ^2.

This formulation was based entirely on the frequency domain (Zames, 1981), suggesting that using the $H\infty$ standard as a measure of performance would better satisfy the demands on applications compared to the LQG control. The author claims that the few robustness properties of the LQG could be attributed to the integral criterion in terms of H_2 standard, criticizing the representation of uncertain disturbances by processes of white noise, often unrealistic.

2. METHODOLOGY $H\infty$

Obtaining an optimal control using $H\infty$ is based on finding a controller that stabilizes a system, minimizing the effects of disturbances in the system. This standard is now used to numerically evaluate the sensitivity, robustness, and performance of the closed loop feedback system controller. The $H\infty$ methodology used in control theory has as main objective to synthesize controllers in order to reach stabilization with guaranteed performance. To use the $H\infty$ methods, a control project expresses the control problem as a mathematical optimization problem, and then finds the controller that resolves that optimization.

The techniques of $H\infty$ control have the advantage of classical control techniques, the ease of applications that involve problems of multivariate systems with cross couplings between the channels (Dorf and Bishop, 2011). The robust control approach using the $H\infty$ controller design is described in the simplest possible terms in order to provide a complete overview of the application area. This project includes updated research and offers theoretical and practical applications including flexible structures, robotics, automotive and aircraft control. (Lin, 2007).

The disadvantages of $H\infty$ include the level of understanding of the mathematics involved needed to successfully implement them and the need for a reasonably good model of the system to be controlled. It is important to note that the resulting controller is only optimized over the prescribed cost function and does not necessarily represent the best controller in terms of the usual performance measures used to evaluate controllers, such as spare time, spent power, etc.

2.1 Mathematical Model of $H\infty$

The term $H\infty$ comes from the name of the mathematical space on which optimization occurs. $H\infty$ is the Hardy space of functions of matrices that are analytic and delimited in the half of the open right part of the complex plane defined by $Re(S) > 0$. In complex analysis, Hardy (or Hardy classes) spaces H^n are certain spaces of holomorphic functions in the disk drive or in the middle of the upper plane (Cruz, 1996).

The $H\infty$ rule is the maximum singular value of the function on that space. This can be interpreted as a maximum gain in any direction and at any frequency for the SISO systems, effectively characterizing the maximum magnitude of the frequency response. The $H\infty$ techniques can be used to minimize the impact of a closed loop perturbation: depending on the formulation of the problem, the impact will be measured in terms of stabilization or performance. However, optimizing robust performance and robust stabilization at the same time is not a very easy task. (Doyle and Stein, 1979).

2.2 Standards for signs and systems

According to (Dorf and Bishop, 2011), linear, time invariant, causal and finite-dimensional systems are considered. In the time domain, an input-output model for this system has the form of a convolution function given by:

$$y(t) = \int_{-\infty}^{\infty} g(t-\tau)u(\tau)d\tau \tag{1}$$

This system has a state space model given by:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad\qquad y(t) = Cx(t) + Du(t) \tag{2}$$

where A, B, C, D are real matrices of appropriate size.
Let $G(s)$ be the transfer matrix of the system given by:

$$G(s) = D + C(sI - A)^{-1}B \tag{3}$$

Another well known notation is the dot matrix notation of the compressed system given by:

$$G(s) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{4}$$

where $G(s)$ is a block matrix.

2.3 Standard for systems

The standard H_p with $1 \leq p \leq \infty$ for multivariate systems is given by a stable transfer function G:

$$\|G\|_p = \left(\int_{-\infty}^{\infty} |G(j\omega)|^p \, d\omega \right)^{\frac{1}{p}} \tag{5}$$

A norma H_∞ é dada por:

$$\|G\|_\infty = \underset{\omega}{sup}\, \sigma_{max} \left| G(j\omega) \right| \tag{6}$$

where σ_{max} is the maximum singular value.

Two well-known performance measures in optimal control theory are the H_2 and $H\infty$ rules defined in the frequency domain by a stable transfer matrix $G(s)$;

$$\|G\|_2 = \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} trace \left[G(j\omega) * G(j\omega) \right] d\omega \right)^{\frac{1}{2}} \tag{7}$$

2.4 Hamiltonian matrix notation

The solution to the $H\infty$ control problem contains algebraic Riccati equations in which the following Hamiltonian matrix notation is introduced to simplify the representation of the EAR solution. Consider the following Riccati equation:

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{8}$$

The solution that stabilizes this equation is given by the matrix P of the Riccati equation as a function of the matrix H, $P = (H)$, where H is represented by:

$$H = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \tag{9}$$

being $(A - BR^{-1}B^T P)$ stable.

2.5 Implementation Methodology $H\infty$

First, the process is represented according to the following standard configuration:

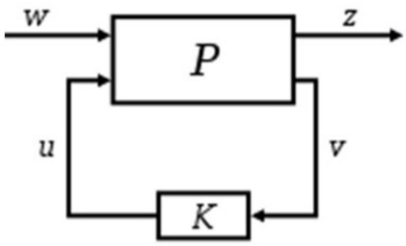


Figure 1- Standard feedback control

In matrix form it is:

$$\begin{bmatrix} z \\ v \end{bmatrix} = P(s) \begin{bmatrix} w \\ u \end{bmatrix} \tag{10}$$

$$= \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{11}$$

$$u = K(s)v$$

The realization in state space of the generalized plant P is given by:

$$P = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$$

Or

$$\begin{cases} \dot{x} = Ax + B_1w + B_2u \\ z = C_1x + D_{11}w + D_{12}u \\ y = C_2x + D_{21}w + D_{22}u \end{cases} \tag{12}$$

The diagram of Figure 1 represents a general description of the robust control system structure in which $P(s)$ is the generalized two-input plant model, the exogenous input $w(t)$, which includes reference signal and disturbances, and the manipulated variables $u(t)$. There are two outputs, the error signals $z(t)$ that we want to minimize, and the measured variables $v(t)$, which we use to control the system, where K is the controller model. $v(t)$ is used in K to calculate the manipulated variable $u(t)$. Note that all these variables are usually vectors whereas P and K are arrays.

Direct manipulations provide the following closed-loop transfer function:

$$T_{zw}(s) = P_{11}(s) + P_{12}(s)[I - K(s)P_{22}(s)]^{-1}K(s)P_{21}(s) \tag{13}$$

The above expression is also known as Linear Fractional Transformation (LFT) of the interconnected system.

In short, the purpose of the robust control is to find a controller that stabilizes $u(s) = K(s)v(s)$ so that $left\,|Tzw(s)\,right\,| < 1$, that is, to minimize the norm:

$$\|T_{z\to w}(P,K)\|_\infty$$

in which the norm is subject to the K controller that stabilizes P internally.

According to (Chen et al., 2007), the small gain theorem tells us that: since $M(s)$ is stable and $\gamma > 0$. The interconnected system shown in figure 2 is well positioned and internally stable for every stable $\Delta(s)$ if the condition of the small gain,

$$\|M(s)\|_\infty \|\Delta(s)\|_\infty < 1 \tag{14}$$

is satisfied.

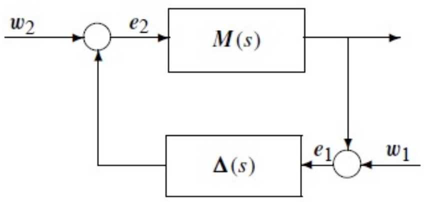


Figure 2- Esquema para o teorema do ganho pequeno.

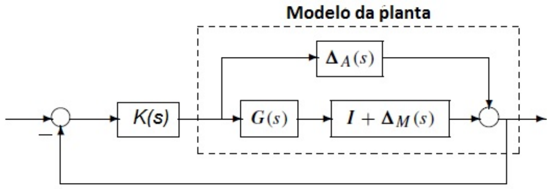


Figure 3- Controle de realimentação com incerteza

2.6 Unstructured uncertainties

According (Skogestad and Postlethwaite, 2007), unstructured uncertainties can be classified into additive and multiplicative uncertainties. The structure of the feedback system with uncertainties is shown in Figure 3;

In general, the uncertainty model can be represented as follows:

$$G_p(s) = \Delta_A(s) + G(s)[I + \Delta_M(s)] \tag{15}$$

in which: if $\Delta_A(s) \equiv 0$, then $Gp(s)$ is a multiplicative uncertainty model; case $\Delta_M(s) \equiv 0$ the model $G_p(s)$ is characterized as an additive uncertainty, with the result that $G_p(s) = G(s) + \Delta_A(s)$ (Doyle and Stein, 1979).

2.7 Generalized model with weighting functions

The diagram shown in figure 4, shows the weighted control structure in which W_1, W_2 and W_3 are the weighted sensitivity functions. It is assumed that $G(s)$, W_1 and W_3 are all them selves, that is, they are bounded when $s \to \infty$, where $z(t) = [z_1, z_2, z_3]^T$ are output vectors. not directly used to construct the control signal vector w_2.

2.8 Optimum controller design $H\infty$

In the optimal controller design $H\infty$, the optimal criterion is defined as follows:

$$\max_{\gamma} \|T_{zw}\| < \frac{1}{\gamma} \tag{16}$$

Or,

$$\max_{\gamma} \begin{bmatrix} W_1 S \\ W_2 k S \\ W_3 T \end{bmatrix} \leq \frac{1}{\gamma} \tag{17}$$

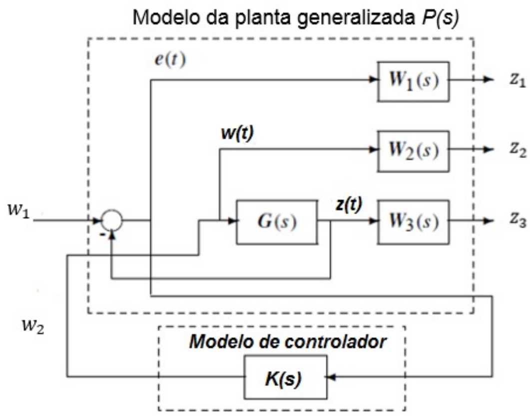


Figure 4- General block diagram of the weighted sensitivity functions

The three above weights can be individually weighted by γ, through interactions with γ variation, thus obtaining the optimal γ^*.

3. RESULTS

The DHM transfer function, was determined by a 3rd order differential equation and placed in the transfer function form and is given by:

$$G(s) = \frac{1}{s(s+5)(s+10)} \tag{18}$$

After simulations performed with MATLAB, we obtained the weighting functions that represent the uncertainties of the system with variation of the parameter ρ:

$$W_1(s) = \frac{100(0.005s+1)^2}{\rho(0.2s+1)^2} \tag{19}$$

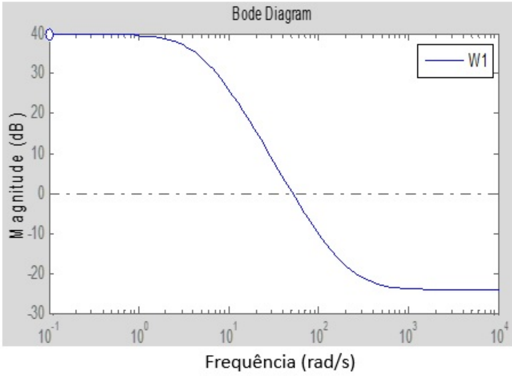
$$W_3(s) = \frac{s^2}{40000} \tag{20}$$

Figures 5 and 6 represent the Bode diagram of the weighting functions $W_1(s)$ and $W_3(s)$, respectively:

From the $hinf(.)$ function of MATLAB, one can design the $H\infty$ controller directly and the generalized two-port plant can be stabilized. The generalized plant controller $G_c(s)$ is obtained in the following form:

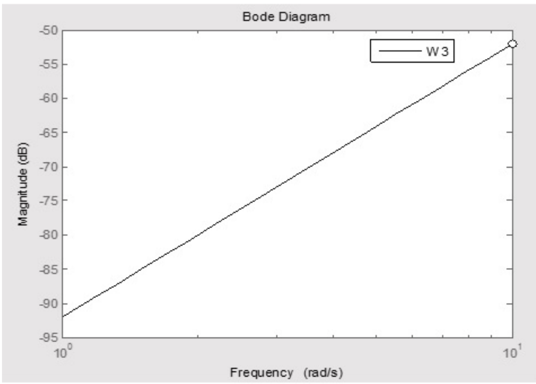
$$G_c(s) = \frac{3.4391(s+10)(s+5)(s+42.84)}{(s+5)^2(s^2+308.5s+4.388)} \tag{21}$$

Figures 7 and 8 show the closed loop system response to different values ρ in the weighting function $W_1(s)$. It is concluded from the graphs that the smaller the value of the parameter ρ the greater the response to the step of the system with reduced overshot, remaining approximately the same form in the curves of the dynamic response.



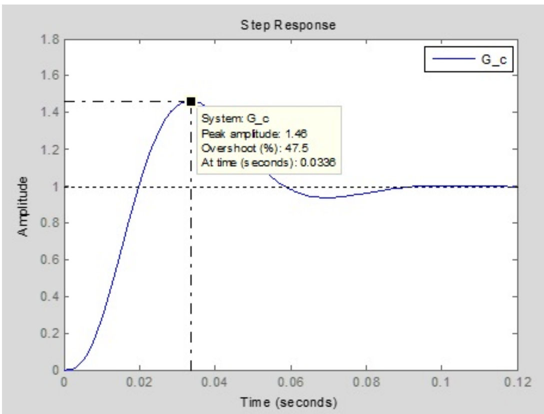
$W_1(J\omega)$

Figure 5- Função de ponderação W_1



$W_3(J\omega)$

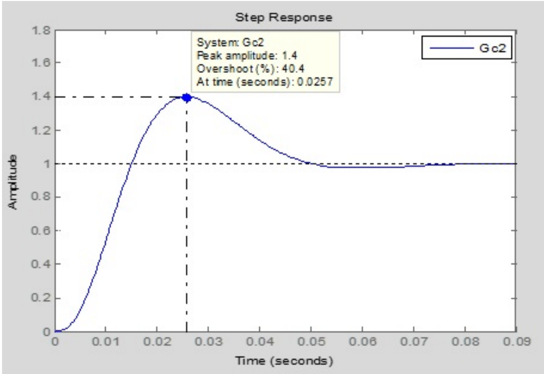
Figure 6- Função de ponderação W_3



$\rho = 1$

Figure 7- Resposta ao degrau para variação de ρ

To ensure the existence of a robust $H\infty$ robust controller for DHM, we use perturbation parameters $\delta \in (-10, 10)$ in the plant model. It can be seen that although the plant model is intensely disturbed, for example, from unstable to stable and with great change in the positions of the poles, the responses to the step are quite close, thus guaranteeing the robustness of the DHM system. When $\delta < 0$, the model of the open-loop plant is unstable. The response to the



$\rho = 0.5$

Figure 8- Resposta ao degrau para ρ

step for the same $H\infty$ controller can be obtained. The optimal $H\infty$ controller is obtained from the γ interaction, as shown in figure 9, obtaining $\gamma^* = 1.2891$ and the optimum $H\infty$ controller designed is:

$$G_{H\infty\textbf{otimo}}(s) = \frac{4.5891(s+55.03)(s^2+10s+50)}{(s+1.496)^2(s+262.5)(s+5)^2} \tag{22}$$

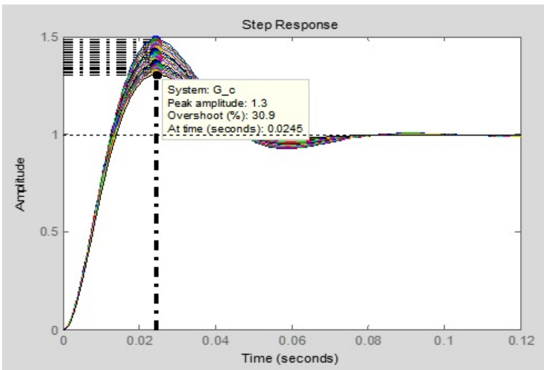


Figure 9- Resposta ao degrau para o $H\infty$ ótimo

4. CONCLUSION

It can be verified that the results for the robust controller $H\infty$ designed for the DHM obtained satisfactory results since they meet the criteria of robustness and satisfy the theorem of the small gain. The decoupling problem of the multivariable control is solved successfully and the performance of the answers is well accepted, respecting, therefore, the robust control requirements, thus allowing a greater applicability of this computational method in other real dynamic systems.

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